Theory, Task Q2

Marker________________________ Student_______________________

TOTAL______________________

N	Statement	Points	Marker		Consensus	
A1	Idea that $\alpha = p_y / p_x$	0.1				
	$p_y = \int F dt$	0.1				
	$F_y = \frac{GMm}{b^2}\cos^3\varphi$	0.2				
	$dt = \frac{dx}{v}$ or $dt = \frac{b\,d\varphi}{v\cos^2\varphi}$, or $p_y = \frac{GMm}{bv}\int_{-\pi/2}^{\pi/2}\cos\varphi\, d\varphi$, or alternative correct equation	0.1				
	Answer $\alpha = \frac{2GM}{bv^2} = \frac{2b_1}{b}$ or k=2	0.25		**(0.75)**		
A2	$\Delta p_x = p(1-\cos\alpha)$ or $\Delta p_x = \mp\frac{\Delta p_y^2}{2p}$	0.1				
	Answer $\Delta p_x = \mp\frac{2G^2M^2m}{b^2v^3} = \frac{2b_1^2}{b^2}p$	0.15		**(0.25)**		
A3	$\Delta N = 2\pi b\Delta b v n\,\Delta t$, up to factor 10	0.2				
	Answer $F_{DF} = \mp 4\pi G^2M^2\frac{\rho}{v^2}\log\Lambda$, up to factor 10	0.2		**(0.4)**		
A4	$\log\Lambda = 7.4\ldots7.6$	**0.2**		**(0.2)**		
A		**1.6**				
B1	$v_{bin} = \sqrt{\frac{GM}{4a}}$	0.1				
	Answer $E = -\frac{GM^2}{4a}$, with correct sign *Incorrect numerical factor doesn't influence following scores.	0.15		**(0.25)**		
B2	$b\sigma = r_m v_0$	0.1				
	$\frac{\sigma^2}{2} = \frac{v_0^2}{2} - \frac{GM_2}{r_m}$	0.1				
	Answer $b = r_m\sqrt{1+\frac{2GM_2}{\sigma^2 r_m}}$	0.3		**(0.5)**		
B3	$(\Delta t)^{-1} = \pi\,\sigma r^2 n$	0.2				
	$r = b_{max}$	0.3				
	$b_{max} = a\sqrt{1+\frac{4GM}{\sigma^2 a}} \approx \frac{2}{\sigma}\sqrt{GMa}$	0.3				
	Answer $\Delta t = \frac{m\sigma}{4\pi GM\rho a}$, up to numeric coefficient	0.2		**(1.0)**		
B4	$\frac{dE}{dt} = -\frac{\pi G^2M^2\rho}{2\sigma}$, up to numeric coefficient	0.15				
	$\frac{da}{dt} = -\frac{2\pi G\rho a^2}{\sigma}$, up to numeric coefficient	0.1		**(0.25)**		
B5	$T_{SS} = \frac{\sigma}{2\pi G\rho a_1}$, up to numeric coefficient	0.7				
	$T_{SS} = 7.3\times10^{-4}\,Gy$, up to factor 10	0.3		**(1.0)**		
B		**3.0**				

C1	$\frac{da}{dt}=-\frac{256}{5}\cdot\frac{G^3M^3}{c^5a^3}$, with correct sign	**0.2**		**(0.2)**		
C2	Integral is calculated $\frac{a_2^4-r_g^4}{4}=\frac{256}{5}\cdot\frac{G^3M^3}{c^5}\cdot T_{GW}$	0.3				
	Answer $T_{GW}=\frac{5}{1024}\cdot\frac{a_2^4c^5}{G^3M^3}$, up to factor 10	0.4				
		0.7		**(0.7)**		
C3	$a_H=0.098\text{pc}$, up to factor 10	**0.1**		**(0.1)**		
C		**1.0**				
D1	$m(r)=\frac{\sigma^2 r}{G}$	0.1				
	$v=\sigma$	0.15		**(0.25)**		
D2	$\frac{dE}{dt}=\frac{dU}{dt}$ *alt. solution:* using angular momentum equation $mv\dot{a}=Fa$	0.3				
	$\frac{dU}{da}=g(a)M$	0.2				
	$\frac{dE}{dt}=-F_{DF}v$	0.15				
	$\frac{da}{dt}=-\frac{GM\log\Lambda}{a\,\sigma}$	0.1		**(0.75)**		
D3	$\text{m}(a)=M$	0.1				
	$a_1=\frac{GM}{\sigma^2}$, up to factor 10	0.1				
	$a_1=1...100\text{pc}$	0.1		**(0.3)**		
D4	$\frac{a_0^2-a_1^2}{2}=\frac{GM\log\Lambda}{\sigma}T_1$	0.4				
	$T_1=\frac{a_0^2\,\sigma}{2GM\log\Lambda}$, up to factor 10	0.25				
	$T_1=0.121Gy$, up to factor 10	0.1		**(0.75)**		
D5	$\dot{E}_{SS}=\dot{E}_{GW}$	0.1				
	$a_2^5=\frac{512}{5}\cdot\frac{G^3M^3a_1^2}{c^5\sigma}$, up to factor 10	0.1				
	$a_2=0.001...0.1\text{pc}$	0.1		**(0.3)**		
D6	Idea of neglecting $\dot{E}_{GW}$ in slingshot stage and neglecting $\dot{E}_{SS}$ in gravitational waves stage	0.25				
	$T_2\approx\frac{\sigma}{2\pi G\rho_1 a_2}$	0.2				
	$T_2\approx10^{-3}...10^{-1}Gy$	0.65				
	$T_3\approx10^{-4}...10^{-1}Gy$	0.65		**(1.75)**		
D7	$T_{ev}=T_1+T_2+T_{GW}$	0.1				
	Scored if D6 score > 0 $\text{T}_{ev}=0.02...2.00\text{Gy}$	0.2		**(0.3)**		
D		**4.4**				
	TOTAL	**10**				