Part A

A1. According to diffraction grating formula

$h \sin \varphi = m\lambda$, where $m = 1$, thus

$$h \sin \varphi = \lambda \quad (1s)$$

A2.

φ, º	θ, º
35	61,5
36	55,5
37	49,5
38	45
39	39
40	35
41	31,5
42	25
43	18,5

A3. Using Bragg-Snell law (2) from the task:

$$2D\sqrt{n^2 - \sin^2 \theta} = m\lambda \quad (2s)$$

and (1s) we get

$$2D\sqrt{n^2 - \sin^2 \theta} = h \sin \varphi$$

$$n^2 - \sin^2 \theta = \left(\frac{h}{2D}\right)^2 \sin^2 \varphi$$

$$\sin^2 \theta = n^2 - \left(\frac{h}{2D}\right)^2 \sin^2 \varphi \quad (3s)$$

$$n = \sqrt{Intercept}, \quad (4s)$$

where intercept is taken from Fig.1

$$D = \frac{h}{2\sqrt{|Slope|}}, \quad (5s)$$

where slope is taken from Fig.1

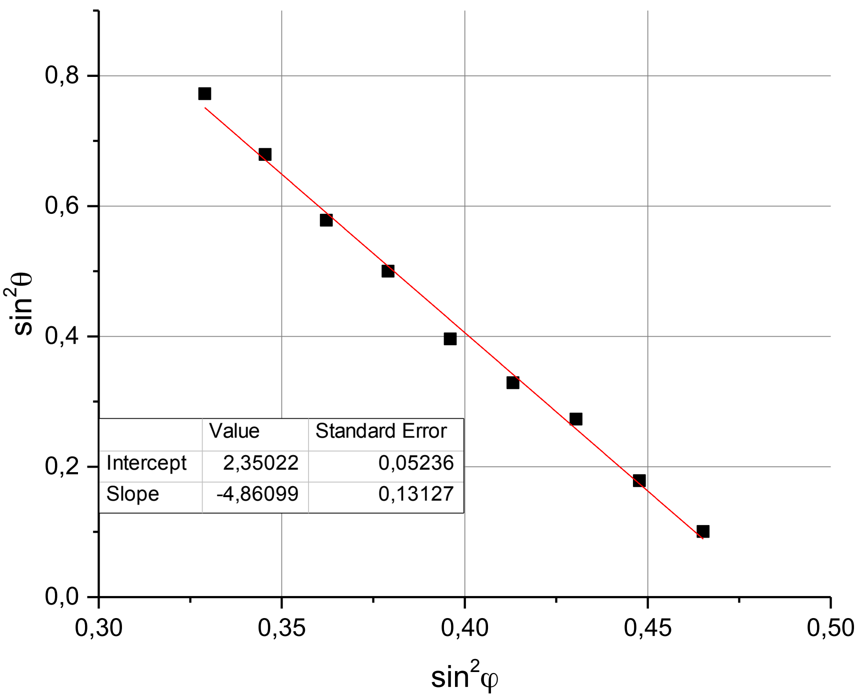

Fig. 1

A4. Taking slope and interception point from the Fig. 1 from (4s), (5s) we get:

$$n_X = 1.53$$

$$D_X = 227\ nm$$

Part B

B1. Minimum can be observed only in the red zone, so we use red laser with wavelength $\lambda = 650\ nm$

B2-B3.

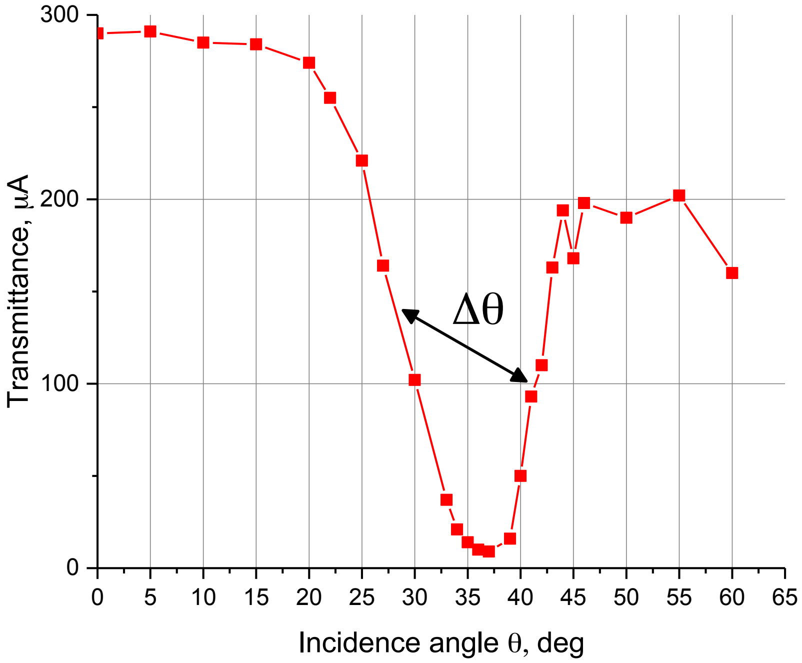

Fig. 2

B4. From the Fig. 2 we find minimum at $\theta_1 = 37^o$ with width of $\Delta\theta_1 = 14^o$

B5. Using (2s) we get

$$2D\sqrt{n^2 - \sin^2\theta_1} = m\lambda;$$

For the normal wavelength we can write

$$2Dn = m\lambda_X;$$

$$\lambda_X = \frac{\lambda n}{\sqrt{n^2 - \sin^2\theta_1}} = 707\ nm. \quad (6s)$$

B6. Using (6s) we determine

$\theta_{min} = 27^o \Rightarrow \lambda_{min} = 683\ nm$

$\theta_{max} = 41^o \Rightarrow \lambda_{max} = 719\ nm$

$\Delta\lambda = \lambda_{max} - \lambda_{min} = 36\ nm$

$$\Delta n = \frac{\pi}{2} n \frac{\Delta\lambda}{\lambda} = 0.12$$

B7. Minimum for the wet sample is at $\theta_2 = 51^{\text{o}}$.

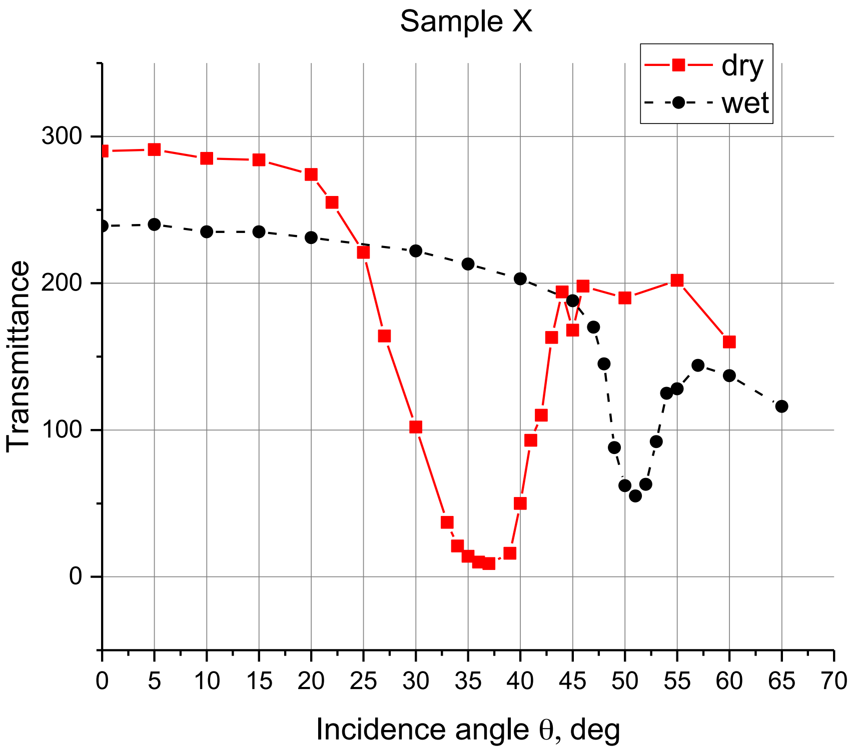

Fig. 3

B8. Using (2s) for n_{wet} and n_{dry} one can obtain

$$2D\sqrt{n_{dry}^2 - \sin^2\theta_1} = \lambda$$

$$2D\sqrt{n_{wet}^2 - \sin^2\theta_2} = \lambda$$

Using (4) and (5) from the task we determine

$$\frac{7}{9}p = n_{wet}^2 - n_{dry}^2 = \sin^2\theta_2 - \sin^2\theta_1$$

$$p = 0.31$$

$$n_{AAO} = \sqrt{\frac{n^2 - p}{1 - p}} = 1.72$$

B9. Taking $\Delta n = 0.12$ and $p = 0.31$ we get

$$n_1 = n + \frac{\Delta n}{2} = 1.59$$

$$n_2 = n - \frac{\Delta n}{2} = 1.47$$

Equation (4) yields

$$p_1 = \frac{n_{AAO}^2 - n_1^2}{n_{AAO}^2 - 1} = 0.22$$

$$p_2 = \frac{n_{AAO}^2 - n_2^2}{n_{AAO}^2 - 1} = 0.41$$

Part C

C1. For the normal incidence ($\theta = 0$) we can observe three minima at angles $\varphi_1 = 43°, \varphi_1 = 35°,\ \varphi_1 = 29°$, therefore $\lambda_1^{sp} = 682\ nm, \lambda_2^{sp} = 574\ nm, \lambda_3^{sp} = 485\ nm.$

C2.

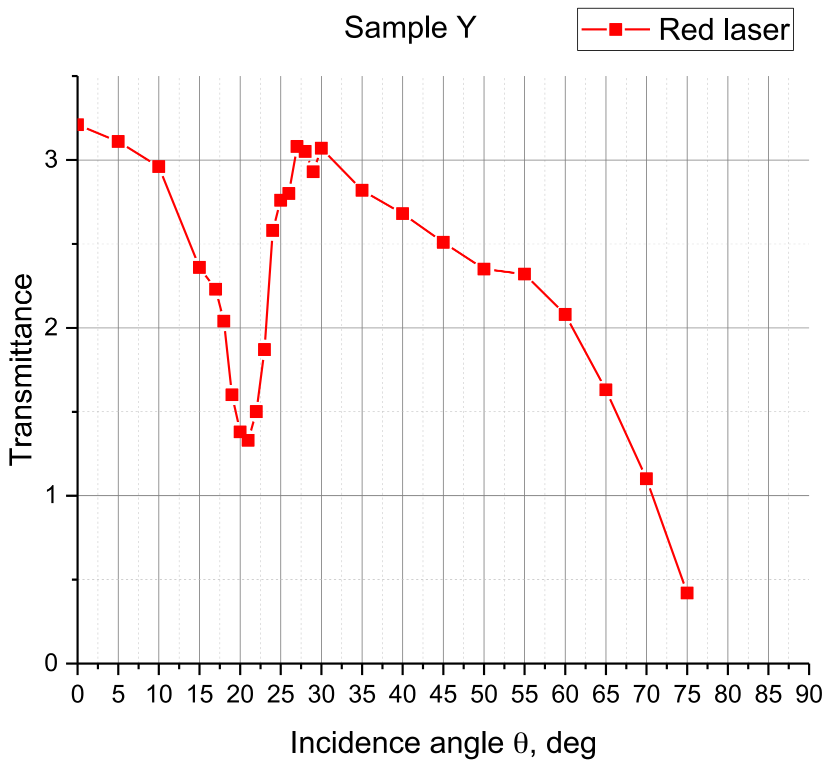

Fig. 4

C3.

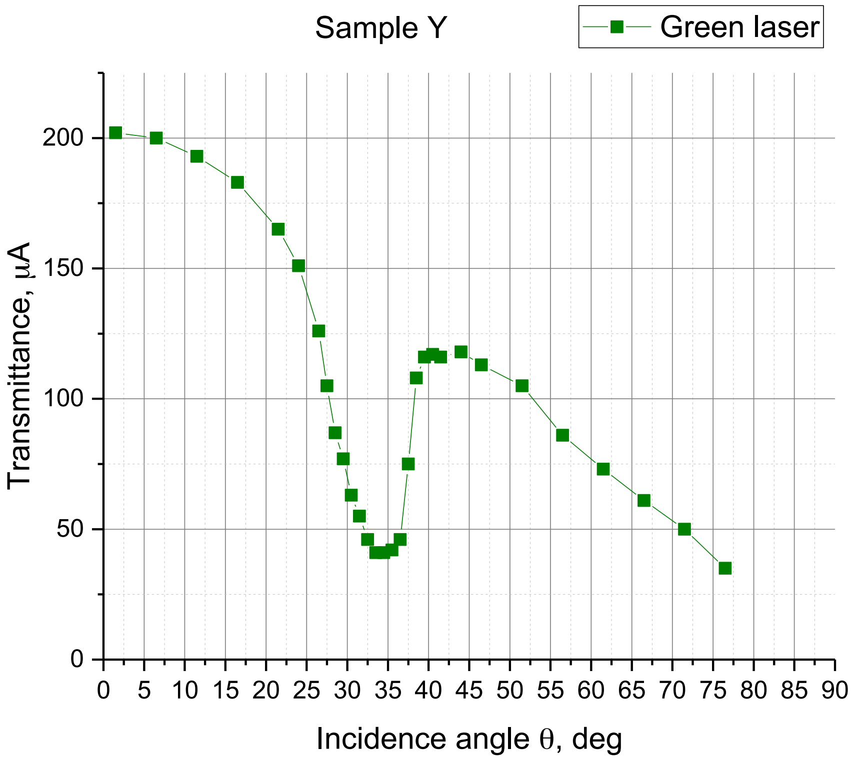
Sample Y
Green laser
Transmittance, μA
Incidence angle θ, deg
0
50
100
150
200
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90

Fig. 5

C4.

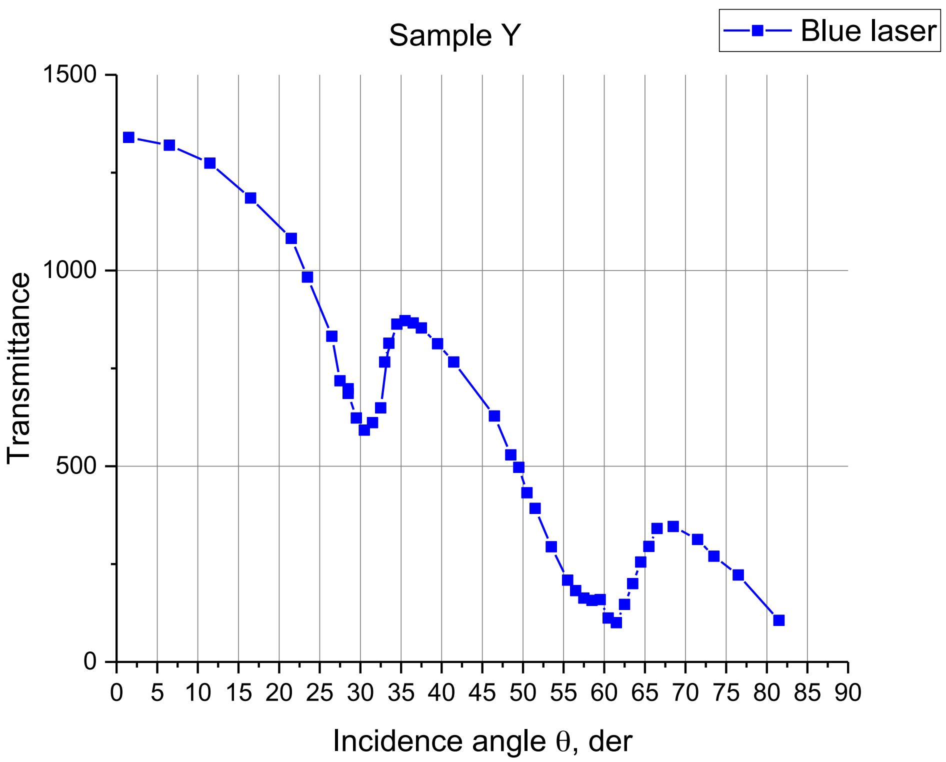
Sample Y
Blue laser
Transmittance
Incidence angle θ, der
0
500
1000
1500
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90

Fig. 6

C5. Using (6s), we calculate normal wavelengths for an arbitrary order $m' = m + const$

$$\lambda_Y = \frac{\lambda n}{\sqrt{n^2 - \sin^2\theta_1}}$$

λ, nm	$\theta,\ deg$	$\lambda_Y,\ nm$	m'
659	21	678	1
530	34	569	2
400	57	478	3
400	30	423	4

C6. According to (2s) dependence $m'\left(\frac{1}{\lambda}\right)$ should be linear:

$$2Dn = (m' - const)\lambda$$

$$m' = \frac{2Dn}{\lambda} + const$$

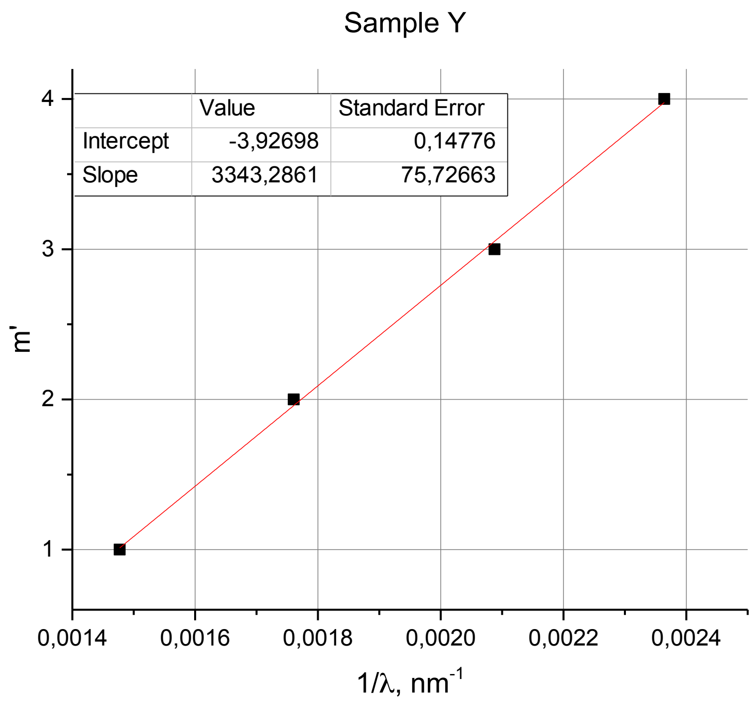

Fig. 7

$$m = m' - Intercept,$$

where intercept is taken from Fig.7

$$Intercept \simeq -4$$

$\lambda,\ nm$	m

677	5
568	6
479	7
423	8

C7.

$$D_Y = \frac{Slope}{2n}$$

where slope is taken from Fig. 7

$$D_Y = 1080\ nm$$

C8. Let I_1 be the half-sum of intensities to the left and to the right from the minimum, and I_2 be intensity in the minimum. Transmittance $t = \frac{I_2}{I_1}$.

$\lambda,\ nm$	t
677	0.42
568	0.29
479	0.22
423	0.63

Part D

D1. One can find up to 6 maximums:

$\lambda_Z,\ nm$
808
696
611
499
462
402

D2. According to (2s) dependence $m'\left(\frac{1}{\lambda}\right)$ should be linear:

$$2Dn = (m' - const)\lambda$$

$$m' = \frac{2Dn}{\lambda} + const$$

Dependence $m'\left(\frac{1}{\lambda}\right)$ will be linear if we take m'=1,2,3,5,6,8 for visible minimums (see fig.8).

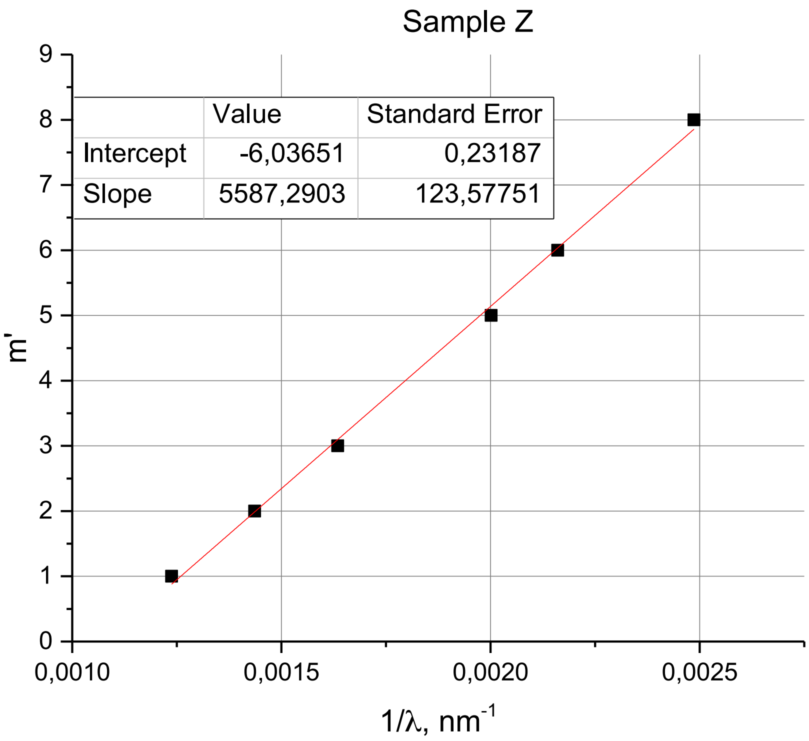

Fig. 8

$$m = m' - Intercept$$

From fig. 8 $Intercept = -6$.

$$m = m' + 6$$

$\lambda_Z,\ nm$	m'	m
808	1	7
696	2	8
611	3	9
499	5	11
462	6	12
402	8	14

D3.

$$D_Z = \frac{Slope}{2n},$$

where slope is taken from Fig. 8

$$D_Z = 1802\ nm$$

D4.

$$m = \frac{Slope}{\lambda} \Rightarrow \lambda = \frac{Slope}{m}$$

Missed minimums correspond to order m=10 and 13:

m	λ_Z nm
10	559
13	430

Part E

E1. We recognize the sample Y by such feature, that 2 central transmittance minimums are deeper than 2 side transmittance minimums. So, it is ***n-6***.

E2. For the sample Z $m = 10$ and $m = 13$ are missing, $m = 8, 9, 11, 12$ are not. So, it is ***hi5-5***.